NON-DURABLE CONSUMPTION VOLATILITY AND ILLIQUID ASSETS[*]

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PRELIMINARY AND INCOMPLETE.

ABSTRACT. We identify five distinguishing characteristics of assets, and develop a life-cycle model of the consumer that incorporates these relevant features. In the specification used for this study, the consumer derives utility from non-durable consumption and stock in a risky asset: housing. An important feature of the model is that the housing adjustment costs are non-convex. These adjustment costs generate lumpy changes in the stock of the risky asset over the life-cycle. The model predicts that consumption volatility is increasing both in the ability to borrow against the assets held in the consumer's portfolio and in the illiquidity of the portfolio. Evidence from the Consumer Expenditure Survey supports this model prediction.

KEYWORDS: Consumption Volatility, Asset Liquidity, Liquidity Constraints, Numerical Methods

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There are two reasons why consumers choose to hold assets: (1) they derive utility from holding the asset, (2) the expected discounted marginal utility from holding the asset for a certain time period is greater than the current marginal utility from consuming. The types of assets they hold are differentiated along five dimensions. First, assets can be solely investment vehicles or they can serve the dual role of an investment vehicle and a consumption good. Second, assets have different expected returns. Third, assets have different risk profiles. Fourth, the degree to which an asset may be levered is not uniform. Finally, the liquidity of assets differs across asset classes and even within asset classes.

This paper investigates the influence that asset liquidity and the degree to which an asset may be levered have on nondurable consumption and portfolio choice in a life-cycle setting. Portfolio choice models often assume that an agent may switch costlessly between risky and risk-free assets. Samuelson (1969) and Merton (1971) are the two seminal examples. Their assumption that markets are complete and the two-fund separation theorem imply that agents can satisfy mean-variance preferences in each period.

In the data, however, there is evidence that agents are not always able to costlessly alter their allocations of risky and risk-free assets. Heaton and Lucas (1997) and Cocco, Gomes, and Maenhout (2002) note that market incompleteness hinders investors from insuring against labor income risk. Further, they conclude that under certain conditions, labor income acts as a substitute for risk-free asset holdings. Beyond market incompleteness, transitioning from one financial asset to another often requires a brokerage fee. Yet there are still many cases, such as IRAs, in which the brokerage fee is minimal or even nonexistent. Campbell (2007) reports that a majority of public equity held by investors is in mutual funds or retirement accounts.

Even in cases in which the brokerage fees are trivial, the agent faces a cost of acquiring the information necessary to make an optimal financial decision. For the more financially astute agents, the cost is merely the time it takes to acquire information and analyze the decision. However, for those less confident in making financial decisions, the aid of financial consultants is often necessary. Indeed, Calvet, Campbell, and Sodini (2007) find that poor and less educated households are more likely to make "investment mistakes" than wealthier, higher educated households. They characterize "investments mistakes" as: (1) Nonparticipation in risky asset markets, (2) Underdiversification of risky portfolios, and (3) Failure to exercise options to refinance mortgages. Whatever the reason may be, investor behavior provides some evidence that there exist barriers to participation in asset markets.

To capture barriers to participation in asset markets in this paper, we assume that all financial transactions in risky markets incur strictly positive transaction costs. The size of the transaction cost in the context of different asset classes is debatable, and for that reason we will focus on housing in this paper. With respect to fixed transactions costs, housing adjustment costs likely represent the upper limit of those incured by a typical consumer in his or her life. Given the assumption about the existence of fixed transaction costs, agents are unable to optimally rebalance their portfolios in each period. One of the goals of this work is to gain a better understanding of these non-convex transactions costs affect nondurable consumption.

The model predicts that consumption volatility is decreasing in the liquidity of the asset. We also find that consumption volatility increases as the ability to borrow against the asset increases. Large fixed transaction costs cause there to be lumpy adjustment in the stock of the asset. When this effect is combined with borrowing constraints, it causes the agent's consumption to become very volatile around the adjustment points. This effect is discussed further in section 4 of the paper. We find evidence in consumption data retrieved from the Consumer Expenditure Survey to support the prediction of the model.

In context, this work most closely relates to that of Grossman and Laroque (1990) and Flavin and Nakagawa (2008). Grossman and Laroque introduce non-convex transactions costs, and show that even with the transaction costs it is optimal for the consumer to hold a mean-variance efficient portfolio at all times. Further, they find that consumers appear to be more risk averse after purchasing a house, and less risk averse before purchasing the house. Flavin and Nakagawa extend the model of Grossman and Laroque to include housing in the utility function. They also allow the price process of the house being sold to differ from the price process of the house being purchased.

The model used in this paper is most closely related to models used by Cocco (2005) who studies portfolio choice over the life-cycle, Yao and Zhang (2005a,2005b), and Li and Yao (2007) who study the life-cycle effects of housing tenure. Yao and Zhang allow renting in their models, and they find that the adjustment cost generates a no adjustment region for housing. They also find that the investor holds more liquid portfolios when he or she is near the boundary of the adjustment region. Li and Yao find that non-housing consumption is less responsive to changes in housing wealth when the transaction cost is included. On the theoretical side, Chetty and Szeidl (2004) and Vereshchangina (2007) examine the effect of consumption commitments in dynamic decision models.

Section 2 of the paper lays out the model. Section 3 gives the baseline parameter calibration of the model. Section 4 gives results from solving the model and running Monte Carlo simulations with different values of the parameters governing liquidity and leverage ability. Section 5 presents empirical evidence to support the model. Section 6 concludes, and provides future directions.

1. Model

We formulate a discrete-time, dynamic decision model of a finitely-lived representative agent who derives utility from consuming a nondurable good and from owning a durable asset. For the first T years of her life, the agent receives a deterministic flow of labor income, which she allocates among consumption of the nondurable good, cash savings held in an interest-bearing account, and investment in the durable asset. At the beginning of year $T+1$, the agent retires, makes no further adjustments to her stock of the durable asset, and converts her cash balances into an annuity that will provide her with a fixed annual payment with which to finance consumption of the nondurable good over the remaining N years of her life.

1.1. **Household Problem.**

Entering each pre-retirement period t, the agent observes the current price of the durable asset P_t, her current cash holdings S_t, and her current stock of the durable asset K_t. She then decides how much of the nondurable good to consume C_t and how much stock of the durable asset to purchase or sell A_t, deriving utility

$$u(C_t, K_t) = \frac{(C_t^{\alpha}(K_t + A_t)^{(1-\alpha)})^{1-\beta}}{1-\beta}.$$

Note that the agent's utility function is a constant relative risk aversion (CRRA) function, within which is nested a Cobb-Douglas consumption function. The period marginal rate of substitution between nondurable and durable consumption is $\frac{\alpha}{1-\alpha}\frac{c}{k}$.

In any given period, the agent's allocation decisions are restricted by three constraints. Total addition to (deduction from) savings in period t must be equal to the agent's realized income in period t less the amount she spends on consumption of the nondurable good and the amount she adds to (deducts from) her durable asset holdings in that period. For any changes made to the durable asset holdings, the agent incurs a transaction cost which is a proportion of the total value of the durable asset held entering period t plus a proportion of the value of the durable asset held at the end of the period. When we think of the asset as a house, the transaction cost setup is analogous to requiring the agent pay a transaction cost on the sale of her old house and on the purchase of her new house. From this point forward, we will talk about the durable asset as if it is a house. The budget constraint described here is formalized in the following equation:

$$X_t = Y_t - P_t A_t - C_t - \tau D_t P_t (K_t + (K_t + A_t)).$$

Here, Y_t is the agent's income in period t, X_t is the addition (deduction) to cash holdings in period t, D_t is a discrete choice variable which takes a value of one when the agent decides to change houses and a value of zero when the agent chooses to stay in the same house:

$$D_t = \begin{cases} 1 \quad if \quad A_t \neq 0 \\ 0 \qquad\quad otherwise \end{cases}$$

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This non-convex transaction cost prevents the consumer from making small, regular changes to housing stock. Figure 1 gives a visual depiction of the transaction cost. Given $K_t = 10$ and $\tau = 0.03$, the figure shows the actual amount of the cost for values of A_t ranging from -10 to 10. Because housing transactions typically incur a large fixed cost (from brokerage fees and search costs), non-convex adjustment costs are appropriate. The interpretation behind the above transaction cost is that the agent pays τ times the sale price of the old house plus τ times the purchase price of the new house. It is debatable whether transactions in other asset markets incur fixed costs. In equity and bond markets there are costs associated with acquiring the information necessary to adjust the stock appropriately in these assets. It may be reasonable to view these as fixed costs rather than variable.

[Figure 1]

For simplicity, we assume that the agent's lifetime income stream is deterministic and known by the agent from the beginning of her working life. The agent is allowed to borrow and lend at a rate of r, however she is restricted to only being able to borrow θ proportion of the value of her house. We restrict θ to be zero in period T so that the agent doesn't retire with debt. The agent's borrowing constraint in period t for $t = 1, ..., T-1$ is:

$$X_t \geq -(S_t + \theta P_t K_t).$$

We may think of θ as a measure of the degree to which the asset may be levered. For a typical consumer their housing assets are likely to have a higher θ than their other financial assets.

Finally, the agent is not allowed to be short of housing. Therefore, we invoke a nonnegativity constraint:

$$A_t \geq -K_t$$

1.2. State Variables.

Entering period t the agent observes her relative stock of saving or debt S_t and chooses change this stock by an amount X_t. If the agent has net savings after the choice of X_t, she earns an interest rate r on the savings account holdings. Otherwise, the agent pays the interest rate r on the total debt she holds. The law of motion for savings is formalized as:

$$S_{t+1} = (1 + r)(S_t + X_t)$$

As discussed above, the agent enters period t with some stock of housing K_t. She then decides to remain in the same house, or to move to a new house of size K_t plus A_t. The stock in the house depreciates annually at a rate of γ. Specifically,

$$K_{t+1} = (1-\gamma)(K_t + A_t)$$

The house price P_t follows a first order mean reverting Markov process. The evolution of the logarithm of this price p_t is given by:

$$p_{t+1} = \mu + \phi(p_t - \mu) + \tilde{\epsilon}_{t+1}$$

Here, μ is the long-run average price, ϕ is the mean reversion parameter, and $\tilde{\epsilon}_t$ is i.i.d. Normal(0,σ^2). σ corresponds to the average per-period volatility of the durable asset price process.

1.3. **Retirement.** At the beginning of year $T+1$, the agent retires, makes no further adjustments to her stock of durable asset, and converts her cash balances into an annuity that will provide her with a fixed annual payment with which to finance consumption of the nondurable good over the remaining N years of her life. In particular, for $t = T+1, T+2, \dots, T+N$, $k_t = k_T$ and

$$C_t = \frac{1-(1+r)^{-1}}{1-(1+r)^{-N}} S_{T+1}.$$

1.4. **Dynamic Decision Problem Formulation.** Assuming that the agent maximizes the present value of utility discounted at an annual subjective rate δ, her dynamic decision problem will be characterized by the Bellman equation

$$\begin{array}{lll} V_t(P_t, S_t, K_t) = & \max_{A_t, X_t, D_t} & \{u(A_t, X_t, D_t) + \delta E_t V_{t+1}(P_{t+1}, S_{t+1}, K_{t+1})\} \\ & s.t. & C_t = Y_t - P_t A_t - X_t - \tau D_t P_t (K_t + (K_t + A_t)) \\ & & C_t \geq 0 \\ & & X_t \geq -(S_t + \theta P_t K_t) \\ & & A_t \geq -K_t \\ & & S_{t+1} = (1+r)(S_t + X_t) \\ & & K_{t+1} = (1-\gamma)(K_t + A_t) \\ & & p_{t+1} = \mu + \phi(p_t - \mu) + \tilde{\epsilon}_{t+1} \end{array}$$

for $t \leq T$, subject to the terminal condition

$$V_{T+1}(P_{T+1}, S_{T+1}, K_{T+1}) = u\left(\frac{r}{1+r} S_{T+1}, K_{T+1}\right).$$

In the way the model is specified, the transactions costs are discontinuous at $A_t = 0$. The implication of this is that the Bellman equation will have nondifferentiable points between states in which it is optimal to purchase a new house and those in which it is optimal to stay in the same house. We address this by employing a strategy in which we find two conditional value functions. V_{0t} is the conditional value function given the discrete choice of not adjusting, and V_{1t} is the conditional value function given the discrete choice to adjust housing stock. Specifically, the conditional value functions are:

$$\begin{aligned} V_{0t}(P_t, S_t, K_t) = \quad & \max_{X_t} \quad \{u(X_t) + \delta E_t \max(V_{0t+1}(P_{t+1}, S_{t+1}, K_{t+1}), V_{1t+1}(P_{t+1}, S_{t+1}, K_{t+1}))\} \\ & s.t. \quad C_t = Y_t - X_t \\ & \qquad C_t \geq 0 \\ & \qquad X_t \geq -(S_t + \theta P_t K_t) \\ & \qquad A_t = 0 \\ & \qquad S_{t+1} = (1+r)(S_t + X_t) \\ & \qquad K_{t+1} = (1-\gamma)(K_t + A_t) \\ & \qquad p_{t+1} = \mu + \phi(p_t - \mu) + \tilde{\epsilon}_{t+1} \end{aligned}$$

and

$$\begin{aligned} V_{1t}(P_t, S_t, K_t) = \quad & \max_{X_t, A_t} \quad \{u(X_t, A_t) + \delta E_t \max(V_{0t+1}(P_{t+1}, S_{t+1}, K_{t+1}), V_{1t+1}(P_{t+1}, S_{t+1}, K_{t+1}))\} \\ & s.t. \quad C_t = Y_t - P_t A_t - X_t - \tau P_t((K_t + A_t) + K_t) \\ & \qquad C_t \geq 0 \\ & \qquad X_t \geq -(S_t + \theta P_t K_t) \\ & \qquad A_t \neq 0 \\ & \qquad S_{t+1} = (1+r)(S_t + X_t) \\ & \qquad K_{t+1} = (1-\gamma)(K_t + A_t) \\ & \qquad p_{t+1} = \mu + \phi(p_t - \mu) + \tilde{\epsilon}_{t+1} \end{aligned}$$

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These conditional value functions are related to the Bellman equation by the following function:

$$V_t(P_t, S_t, K_t) = \max\{V_{0t}(P_t, S_t, K_t), V_{1t}(P_t, S_t, K_t)\}.$$

Because this model cannot be solved analytically, I use the numerical techniques discussed in Miranda and Fackler (2002) to find a solution.[1] There are two advantages to using the approach sketched above. First, the optimands of the maximization problem embedded in the conditional value functions will be continuous. Second, the conditional value functions will be

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[1] Details in the Appendix

differentiable. This property enables us to approximate the conditional value functions using smooth Chebychev polynomials.[2]

2. CALIBRATION

Table I summarizes both the general parameter restrictions and the baseline parameterization.

[Table I]

We set the per-period interest rate r to approximately 100 basis points below the average annualized post-World War II real return on U.S. T-bills. The 100 basis point departure from the real T-bill return is meant to capture an asset that is relatively more liquid than the T-bill. The coefficient of relative risk aversion, β, of 1.5 is fairly standard in the literature. In the 2006 survey of consumer finances, it is reported that housing makes up somewhere between 30 and 40 percent of household expenditure. Therefore, we set the nondurable consumption utility weight, α, equal to 0.7. We set the durable asset depreciation rate, γ, equal to 0.03 and the collateral requirement, θ, equal to 0.80. In addition to the parameters in Table 1, there is also a deterministic income process I_t, specified as:

$$Y_t = Y_0(1 + r)^t$$

3. RESULTS

The purpose of that paper is to study the effects that asset characteristics have on consumption decisions. In the following analysis we solve the model specified above using numerical methods. Each time we solve the model, we vary a single parameter. Then we run Monte Carlo simulations to generate optimal paths for agents. In the present study, we are interested in understanding what the model predicts at different levels of asset liquidity (τ) and at different levels of asset leverage (θ).

3.1. **Asset liquidity.**

Liquidity of assets held by consumers is not constant. The relative thickness or thinness of asset markets is affected by business cycles and aggregate shocks. Additionally, individual real estate markets are not all homogeneous in their liquidity. We examine different levels of asset liquidity ranging from no transactions costs to $\tau = 0.05$ to determine if degree to which the intertemporal changes in liquidity and the heterogeneity between consumers portfolio liquidity affects consumption. Note that at a $\tau = 0.05$, say, the 'round-trip' transaction cost of selling the old house and purchasing a new one is ten percent.

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[2] Source: Mario Miranda. Need to determine if there is a paper that must be cited here.

We choose to be somewhat agnostic about what the 'correct' level of transaction cost should be. Indeed, we intend for the adjustment cost in this setting to not only capture brokerage fees but also the relative thickness of the market and the cost of the agent to acquire information about the new asset. In this sense transactions costs likely differ by market, and depend on whether the agent goes from thin market to thin market, thin market to thick market, et cetera.

Figure 2 depicts the relationship between asset liquidity, leverage ability, and nondurable consumption volatility. As can be seen in the figure, consumption volatility is decreasing in the liquidity of the asset. For perfectly liquid assets ($\tau = 0.0$), the average time series consumption volatility for 1000 Monte Carlo draws is 0.178 for the baseline $\theta = 0.8$. When the transaction cost increases, the consumption volatility increases. At the highest transaction cost we explore ($\tau = 0.05$) the average consumption volatility predicted by the model is 0.24. In section 4.3, we will give some intuition about the mechanism driving this effect.

[Figure 2]

3.2. **Asset leverage.** Some assets may be leveraged more than others. For example, it may be easier to borrow using a house as collateral than a portfolio of equities. There are institutional differences between countries and states that cause the degree to which nearly identical or identical assets may be levered to be different. Government regulation of credit markets can directly affect the degree to which an asset may be levered. We examine how degrees of asset leverage ability ranging from $\theta = 0.4$ to $\theta = 1.0$ affect consumption volatility.

Returning to Figure 2, we can see that consumption volatility is increasing in the degree to which the asset may be levered. At the baseline transaction cost of ($\tau = 0.03$), the average time series consumption volatility for 1000 Monte Carlo draws is merely 0.19 at $\theta = 0.4$. Increasing the leverage ability to 100%, a reasonable level in modern U.S. mortgage markets, the average consumption volatility increases to 0.242.

3.3. **The Mechanism.** In order to get some intuition about the mechanism driving the model predictions above, Figure 3 plots the consumption path and housing stock path for three randomly chosen Monte Carlo draws using the baseline calibration.

[Figure 3]

The fixed transaction cost makes it suboptimal for the consumer to adjust in each period. This feature of the model, generates (s, S) bounds on housing stock similar to those in Grossman and Laroque (1990). That is, at any given time the consumer prefers to be at a housing stock level equal to S. However, because there is a high fixed cost associated with adjustment the agent waits until her housing stock has deviated sufficiently far from S to justify incurring the transaction cost. The lower and upper bounds at which the agent will chose to adjust are defined $\underline{s}$ and $\overline{s}$ respectively. On the right hand side of Figure 3, we can see how the agent waits until the asset depreciates a certain amount before adjusting. In each of the draws displayed here, the agent adjusts housing six times during the forty-five year period. When transactions costs are eliminated (not shown here), the agents adjust housing in every period.

On the left hand side of Figure 3, we observe that consumption becomes more volatile around the adjustment points. This is driven by the liquidity constraints that the consumer faces around the purchasing points. Under convex adjustment costs, these liquidity constraints would not bind as often because the agents are able to accumulate housing stock gradually over time. The liquidity constraints also bind early in the life-cycle as the consumer attempts to smooth consumption. However, liquidity constraints binding early in the life-cycle are not unique to this model and are not related to the specification of the transactions costs. The utility specification dictates that durable and non-durable consumption are nonseparable. Bernanke (1985) notes that this nonseparablility in the utility function causes spillover effects between non-durable and durable consumption. Indeed, we observe these spillover effects. When the consumer enters a period in which it is optimal to adjust housing, non-durable consumption falls. This is the result of the consumer spending a larger proportion of her budget on durable consumption in that period. It is the combination of the non-convex transactions costs, liquidity constraints, and the nonseparable utility function that drive the results above.

4. Data

Given limitations of the data, finding conclusive tests of the model predictions is a challenge. We have settled upon an exercise in which we separate out nondurable consumption of homeowners from that of non-homeowners in the Consumer Expenditure Survey (CEX). We separate the homeowners from the non-homeowners using two methods. First, we us a grouping estimator technique similar to that posed by Browning, Deaton and Irish (1985). This method utilizes demographic data to identify groups of 'likely' homeowners. Second, we use actual response data from the CEX to identify which households own homes. In a final exercise, we separate out nondurable consumption from homeowners who do not own equities from those who do.

After the delineations are made, we examine the time series properties of the consumption of the groups. I have acquired the data extracts of the CEX from the National Bureau of Economic Research (NBER) website. These extracts were assembled by Ed Harris and John Sabelhaus. This data is a pseudo panel spanning 1981 to 2001, and contains detailed information on consumption of households. More details about the data are provided in Appendix A. Table II provides a description of the consumption categories used in this study.

[Table II]

The categories were chosen specifically to capture nondurable household consumption. For this reason, we don't include furniture, appliances, housing, or car purchases. The reader is encouraged to take note that the category car only includes maintenance costs and fuel costs for automobile transportation. Although the variables used in this study are not exhaustive, we believe they provide a reasonable sample of the representative household's nondurable consumption bundle. Other studies, such as Mankiw and Zeldes (1991) look only at food consumption spending from the Panel Survey of Income Dynamics. Attanasio, et al. (2002) look at total nondurable consumption from the British Family Expenditure survey. Their conclusions were that consumption is more volatile for stockholders than for non-stockholders.

Taking consumption growth rates to be the log difference of consumption from one year to the next, we define consumption volatility to be the standard deviation of time series of consumption growth rates. Formally, given a consumption series C_t we let the natural logarithm be denoted as c_t. Then the growth rate of consumption in any period is:

$$\Delta c_t \equiv c_t - c_{t-1}$$

The consumption volatility is just the standard deviation of the vector $\Delta \mathbf{c}$.

4.1. **Analysis of 'Likely' Homeowners' Consumption.**

In this section, we identify groups 'likely' homeowners in the CEX using demographic characteristics. This first step in this process is to build a model that predicts the likelihood that a household owns a home conditional on some demographic characteristics. For this purpose, we estimate a probit model from pooled data set consisting of the 1983, 1986, 1989, 1992, 1995 and 1998 Surveys of Consumer Finances (SCF). The dependent variable measures whether the observation is that of a homeowner or not, and thus is discrete. We relate this variable to demographic characteristics of the household and household head. The probit estimation results are given in Table III.

[Table III]

We use the coefficient estimates from the probit model combined with demographic variables from the CEX to calculate the 'likelihood' of households in the CEX owning a home. We classify households with a likelihood greater than 0.7 as being homeowners and households with a likelihood less than 0.7 to be non-homeowners. Our grouping estimator is calculated by taking the average of consumption in each category for the non-homeowners and homeowners respectively in each year. Then we calculate the consumption volatility for the time series vectors as discussed above. The results are reported in Table IV.

[Table IV]

The results in Table IV show that consumption volatility is higher in each consumption category for the predicted homeowners than it is for the predicted non-homeowners. While these results are comforting, we have enough data within the CEX to determine actual homeownership at the time of the survey. There are a couple reasons why the grouping estimator method used to construct the results in Table IV may not be robust. First, the probit model only explains approximately twenty-one percent of the variance in the discrete homeownership variable according the the McFadden R^2. Second, the threshold of 0.7, at which we set the cutoff between homeowners and non-homeowners, can arguably be set higher or lower. For these reasons, we examine data of actual homeowners in the next section.

4.2. **Analysis of Actual Homeowners.**

[Table V]

Table V was generated by using house value data provided in the CEX extract data set. If the reported house value for a household is greater than zero, we code that household as a homeowner. This table provides some evidence that consumption volatility is higher for homeowners than for non-homeowners. In three of the five categories (Food, Utilities, and Clothes), the consumption volatility is higher for homeowners than for non-homeowners. Of the other two categories, the volatility of consumption on health care services and products is substantially lower for homeowners than non-homeowners. Using data from the 2005 CEX, we find that the weighted average of consumption volatility of the five categories is slightly higher for homeowners than non-homeowners.

This exercise appears to, at the very least, not contradict the model results in section 4. However, the second exercise provides better evidence for the central finding in section 4: that consumption volatility is higher for consumers who hold less liquid assets that may be levered to a high degree. For the most part, equities are more liquid than houses. Samples of intra day trade data provide evidence that large-cap equity common stock is traded multiple times per minute, while small-cap equity data may be traded as infrequently as weekly. Houses often stay on the market for weeks or even months before the market settles on a price. Also, for a typical household, houses are easier to borrow against than equity portfolios. Therefore a test case might be to separate out the households who just own houses from those who just own equities. Unfortunately, the set of households that own equity buy not houses is empty for a majority of our sample period. Therefore, we separate the set of homeowners into two groups: those who own equities and those who do not.

We can think of the household portfolio as being a weighted average of all the assets it owns. The properties of the household portfolio (liquidity, leverage ability, expected return, etc.) can similarly be taken to be a weighed average. Consider two households (A and B) whose portfolios are of equal value, but household A holds equities and a house while household B only holds the house. The liquidity household A's asset portfolio will be higher than that of household B. Likewise, the ability ability for household A to lever its portfolio will be less than household B. Therefore, ceteris paribus, we propose that households who own houses and equities will have lower consumption volatility than households that only hold houses. This provides the framework for the exercise carried out in Table VI.

[Table VI]

To construct the groups of stockholders and non-stockholders in Table VI, we start with the set of homeowners used in Table V. Within that set, we define households as stockholders if the securities variable in the CEX Extract data set is greater than zero. For four of the five consumption categories, consumption volatility is higher for non-stockholders than for stockholders. The consumption of utilities is the one exception, with stockholders utility consumption volatility being higher than non-stockholders. Using data from the 2005 CEX, we find that the weighted average of consumption volatility of the five categories is higher for non-stockholders than stockholders. This is in line with the model prediction that consumption volatility is higher for households that hold less liquid, highly leveragable asset portfolios.

5. Conclusion

This paper presents a model that explains stylized facts about consumption volatility. Namely, that consumption volatility is higher for consumers who hold asset portfolios that are less easily levered and less liquid. The mechanism that drives the predictions of the model is rooted in non-convex adjustment cost which creates regions of inaction with respect to portfolio adjustment within the agent's state space. When these non-convex transactions are combined with the liquidity constraints that arise from secured borrowing requirement, household consumption is very volatile around adjustment points. This volatility in consumption arises from the liquidity constraints binding around the adjustment points.

The model is flexible to the extent that it allows one to vary the parameters governing the liquidity (τ), leverage ability (θ), and volatility (σ) of the asset. The expected return of the asset can be changed by allowing the long run average price parameter (μ) to be time-varying.

One can move the asset in and out of the utility function simply by changing the Cobb-Douglas weight on nondurable consumption (α) to one. In short, although this study specifies the risky asset to be a house, the model presented in this paper has the ability to examine all five characteristics of assets identified in the introduction.

Future work on this topic will explore how correlation between stochastic labor income and risky asset returns affect portfolio choice and consumption decisions in the life-cycle. Another offshoot of this project is to take the partial equilibrium model presented here to a general equilibrium setting with many heterogeneous agents. By specifying a general equilibrium model with heterogeneous agents, we will be able to endogenize the asset price process. Endogenous asset prices are more reasonable than exogenous asset prices in both household portfolio choice models with housing and in institutional portfolio choice models with small-cap equities, over-the-counter derivatives, and other assets that trade in thin markets.

Appendix A. Data Appendix

The CEX Interview Survey is a pseudo panel that follows households for four quarters, and asks questions about some 600 income and expenditure categories. The CEX Extracts, assembled by Ed Harris and John Sabelhaus, are available for download on the NBER data website. The purpose of the extracts is to make the data more accessible by eliminating noise and combining some of the consumption categories. In the household data, the four quarterly files are merged into one annual file. Many questions within the survey are asked about 'The previous year's...', and therefore it simplifies things to aggregate quarterly variables up to the annual level. Of course the downside to doing this is the loss of information.

The annual files, ranging from 1981 to 2001, also contain demographic information about the households, asset holding information, and other income information. We eliminate households that did not respond in all four quarters, and households for the head of the household did not directly respond to the survey. The average sample size after the eliminations for the 1981 to 2001 period is 2191.76.

For the first exercise above, we separate out the homeowners from the non-homeowners. The average number of homeowners over the twenty year sample is 1173.43, while the average number of non-homeowners is 1018.33. So, homeowners comprise on average 53.54 percent of the sample.

In the second exercise, we take the set of homeowners and separate out the stockholders from the non-stockholders within that set. The average number of stockholders over the twenty year period is 324.29 or 27.6 percent of the homeowners.

Appendix B. Technical Appendix

B.1. **Model Solution.** We solve this finite horizon dynamic programming problem via backward recursion from time T to the beginning. Specifically, in period t the agent chooses a vector $\mathbf{x} = [X_t \;\; A_t]'$ to maximize conditional value functions V_t^i for $i = 0, 1$ a state vector $\mathbf{s}$ and approximation coefficients $\mathbf{a}$:

$$V_{it}(\mathbf{s}) = \max_x \left\{ u_t(\mathbf{x}, \mathbf{s}) + \beta \int \hat{V}_t(\mathbf{s}'; \mathbf{a}) dF(\mathbf{s}'|\mathbf{s}, \mathbf{x}) \right\}$$

We use the collocation method to approximate the unknown functional equation $\hat{V}$ using a linear combination of known basis functions. For this application, we use spectral methods to approximate the function using N Chebychev polynomials evaluated at N Chebychev nodes within the three dimensional state space. Rivlin's Theorem states that Chebychev-node polynomial interpolants are nearly optimal polynomial approximants. To pin down the polynomial coefficients, we require that the value function approximant equal the actual value function at the N state nodes. Therefore, we seek to solve a system of N nonlinear equations in the N unknown coefficients.

We map the conditional value functions into the Bellman equation by evaluating the following relationship at each of the N collocation nodes.

$$V_t = \max \left\{ V_{0t}(\mathbf{s}, x_{0t}^*), V_{1t}(\mathbf{s}, x_{1t}^*) \right\}$$

Here, x_{0t}^* and x_{1t}^* are the vectors of choice variables that maximize the respective conditional value functions given state $\mathbf{s}$.

An issue with this type of model, where we both allow the consumer to accumulate savings and stock in the risky asset and approximate the value function using polynomials, is that we must allow the state space to grow over time. The curse of dimensionality implies that we must be judicious in the number of collocation nodes/basis functions that we use to approximate the value function. The tighter we are able to keep the state space, the better approximation we can achieve with a limited number of nodes. For this reason, we don't want to set the boundaries state space in every period t to be at the absolute maximum and absolute minimum of period T. Therefore, the size of our state space will be time dependent. This poses a problem because the boundaries of the state space must be set ex ante. If the state space is too small, we risk evaluating the polynomial interpolant at points beyond the predesignated boundaries. Chebychev polynomial approximants are known to behave poorly when the state variables extrapolate beyond their bounds. Setting the size of the state space ex ante is cumbersome. However, the finite horizon setting allows us to use this method rather than restricting our parameters to meet the sometimes economically implausible 'impatience constraints.'

B.2. **Simulation.** The solution to the model generates to matrices of control variables at the prespecified state nodes. We simulate M individual 'agent' paths by randomly generating M T-by-1 vectors of house price shocks. We evaluate the evolution of our state periods in each time period, yielding a vector of evaluated states $\mathcal{F}_t \equiv [\tilde{P}_t \ \tilde{K}_t \ \tilde{S}_t]$. Typically, we would interpolate between the nodes to find the control value relating the evaluated states $\mathcal{F}_t$. However, because of the presence of the discrete choice, interpolation between the three state points $\mathcal{F}_t$ yields incorrect results. A second best approach is adopted. In this approach, we chose the control value by identifying the set of prespecified state nodes that are closest to the evaluated states.

Unfortunately, the approach adopted here generates quite a bit of noise. However, we can reduce the noise to an arbitrarily small number by simply incereasing the number of prespecified state nodes. Because we are making comparisons between different calibrations of the same model while using the same simulation approach, we believe that the noise generated by our simulation approach does not influence the main result of this paper.

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Figure 1: Transaction Costs

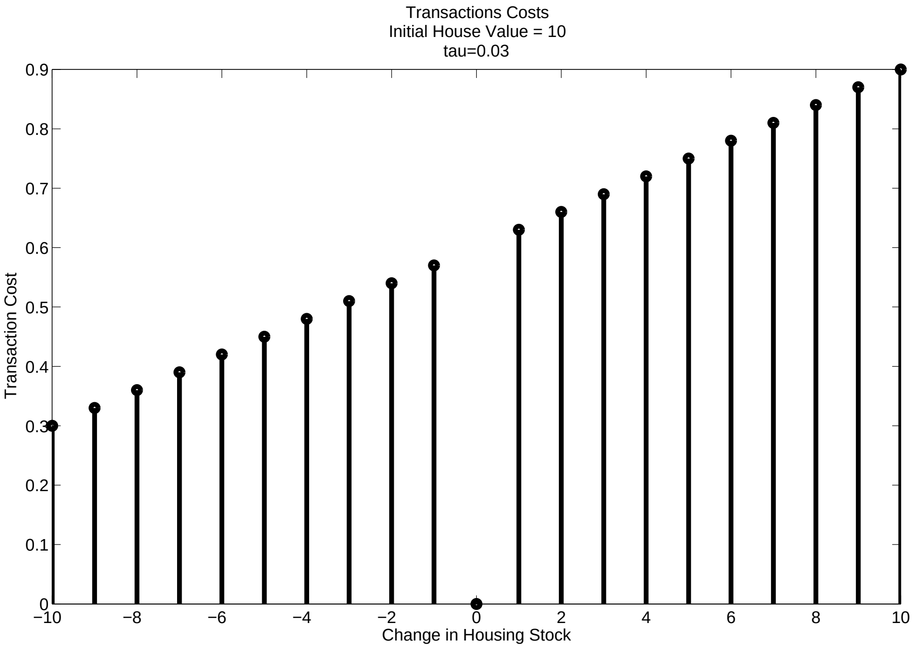

Figure 2: Consumption Volatility in Liquidity and Leverage

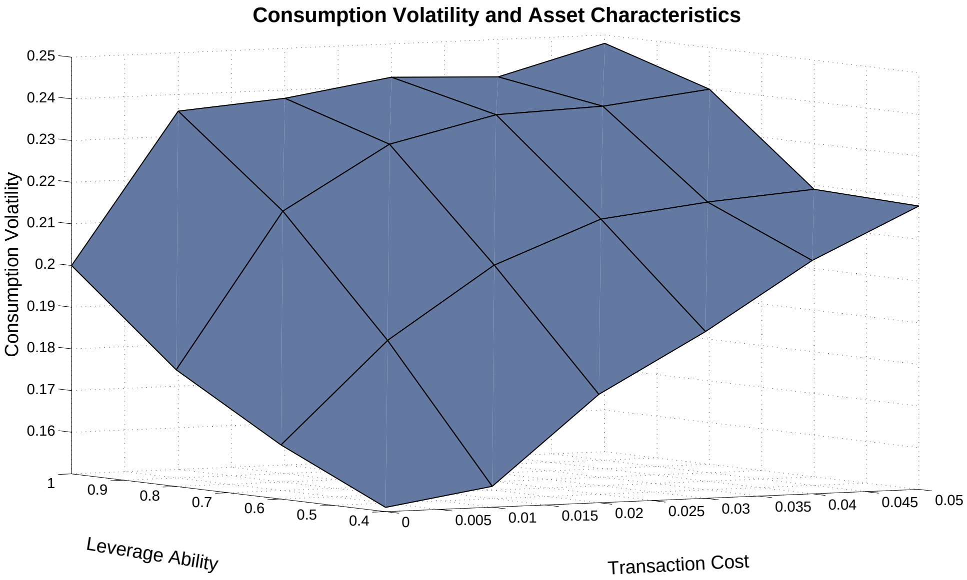

Figure 3: Consumption and Housing Stock

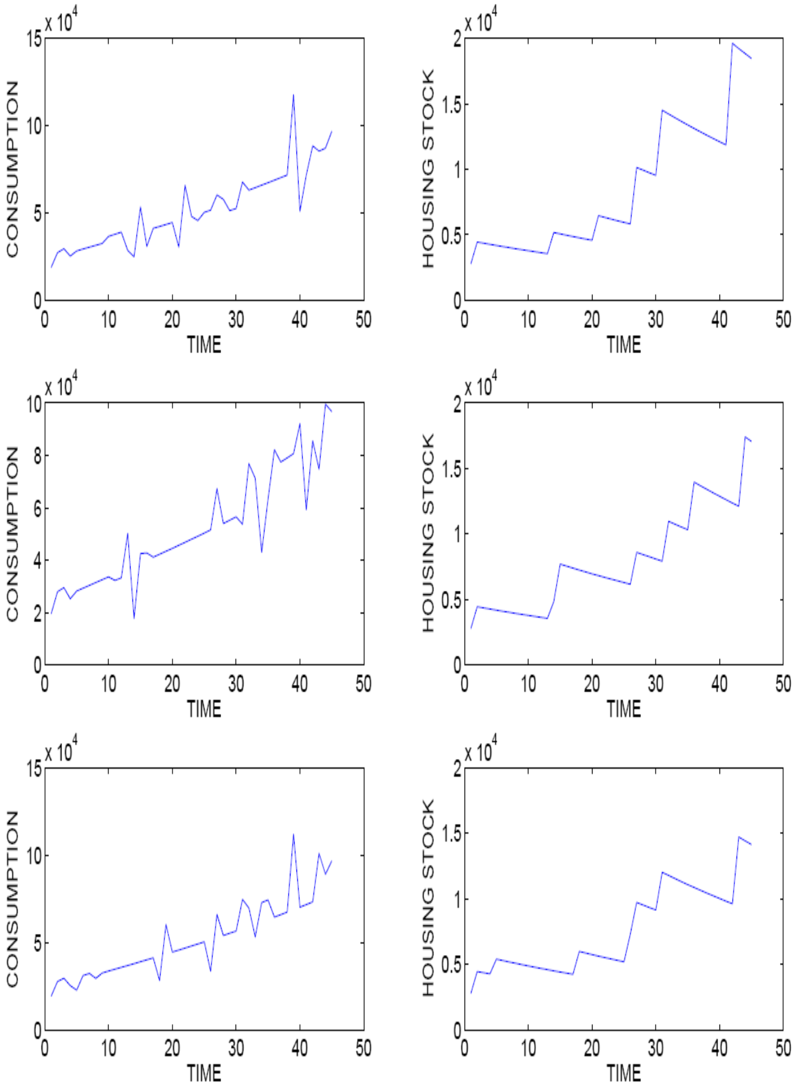

Table I

Parameter	Description	Restriction	Parameterization
r	per-period interest rate	$0 \leq r$	0.02
δ	subjective discount factor	$0 \leq \delta \leq 1$	0.97
γ	durable asset depreciation rate	$0 \leq \gamma \leq 1$	0.02
θ	collateral requirement	$0 \leq \theta \leq 1$	0.80
α	nondurable consumption utility weight	$0 \leq \alpha \leq 1$	0.70
β	coefficient of relative risk aversion	$0 \leq \beta$	1.5
μ	long-run mean return	$0 \leq \mu$	1.0
ϕ	mean reversion parameter	$0 \leq \phi \leq 1$	0.90
σ	standard deviation of durable asset price process	$0 \leq \sigma$	0.06
τ	transaction cost	$0 \leq \phi \leq 1$	0.03
T	number of time periods	$0 \leq T$	45.0

Table II

Category	Description
Food	Includes expenditures on alcohol, tobacco, food consumed at home, work, and in restaurants.
Utilities	Includes expenditures on electricity, gas for home, water, and telephone.
Clothes	Includes expenditures on clothes, tailors, jewelry, and personal care products.
Car	Includes expenditures on car servicing, gasoline, parts, and auto insurance.
Health	Includes expenditures on drugs, doctor visits, hospital visits, health insurance, and orthopedic products.

Table III

VARIABLE	COEFFICIENT ESTIMATE	STANDARD ERROR
DEPENDENT VARIABLE: OwnHouse=1		
Constant	-4.7277	0.09132
AgeHead	0.1266	0.00345
AgeHead^2	-0.0009	0.00003
NumPeople	0.1925	0.00761
White	0.5590	0.02535
HighSchool	0.0703	0.02075
College	0.3093	0.02209
Male	0.5419	0.02370

Table IV

CATEGORY	PERCENT OF 2005 EXPENDITURES	CONSUMPTION VOLATILITY	
		HOMEOWNERS	NON-HOMEOWNERS
Food	14.75	0.0275	0.0199
Car	8.98	0.0393	0.0271
Utilities	6.89	0.0551	0.0270
Health	5.92	0.0993	0.0373
Clothes	5.31	0.0796	0.0548

Table V

CATEGORY	PERCENT OF 2005 EXPENDITURES	CONSUMPTION VOLATILITY	
		HOMEOWNERS	NON-HOMEOWNERS
Food	14.75	0.0238	0.0228
Car	8.98	0.0291	0.0297
Utilities	6.89	0.0309	0.0258
Health	5.92	0.0442	0.0547
Clothes	5.31	0.0640	0.0501

Table VI

CATEGORY	PERCENT OF 2005 EXPENDITURES	CONSUMPTION VOLATILITY	
		NON-STOCKHOLDERS	STOCKHOLDERS
Food	14.75	0.0297	0.0290
Car	8.98	0.0413	0.0390
Utilities	6.89	0.0294	0.0391
Health	5.92	0.0624	0.0507
Clothes	5.31	0.0818	0.0683